I–V characteristics of an SIS Tunnel Junction Under Irradiation of Millimeter Waves

Tutorial for the Practical Course M

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1 Introduction

The current-voltage relation of superconducting tunnel junctions is different from the linear current–voltage characteristic (I–V characteristic) known from normal metal (NIN) tunnel junctions.

- An extreme nonlinearity occurs in a voltage interval of only a few 100 microvolts. The behavior of an ideal SIS tunnel junction is almost comparable to that of a switch.
- Upon coupling of high frequency alternating currents (here at 345 GHz), distinctive steps appear in the current–voltage characteristic, the so-called photon steps.

The subject of this experiment is the analysis of the I–V characteristic of a superconducting tunnel junction with and without irradiation of millimeter waves as well as on application of a magnetic field. Conclusions on the physical properties of a superconducting tunnel junction can be drawn from an analysis of the I–V characteristic.

Security advice: Since liquid helium is used in this experiment, utmost care must be taken during the whole experiment. Both the instructions given by the assistant and this tutorial are to be followed.

1.1 Preparatory questions

Before performing the experiment, you should be prepared to answer the following questions. In addition to this tutorial, the references given in section 1.2 will also be useful for your preparation.

- What is the energy gap of a superconductor?
- How can it be experimentally verified?
- Sketch the I–V characteristic of the tunnel current $I(U)$ of an NIS tunnel junction (normal metal–insulator–superconductor) for T=0 K and explain it in terms of the "semiconductor model" for superconductors.
- Sketch the I–V characteristic of an SIS tunnel junction and explain its behavior at T=0 K and $0<T<T_C$ in terms of the "semiconductor model" for superconductors.
- Why is $eU_{gap}=2\Delta$ for SIS tunnel junctions?
- What is the essential difference between the I–V characteristics of an NIS and an SIS tunnel junction?
- Under which conditions may Cooper pairs tunnel through the barrier of an SIS element?
- What does the supercurrent depend on (first Josephson equation)?
- Derive the magnetic field dependence of the Josephson effects.
- Does the Josephson current really become zero for $\Phi=n\Phi_0$?
- What is the effect of high frequency radiation on the tunneling process of the quasiparticles and how does it show in the current–voltage characteristic of a tunnel junction?
- What are Shapiro steps? What are they caused by?
- Comment on possible applications of SIS tunnel junctions.

1.2 References

General literature on superconductivity:

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TUCKER, J. R.; FELDMAN, M. J.:
"Quantum detection at millimeter wavelengths", Reviews of Modern Physics, Vol. 57, No.4, October 1985, pp.1055-1113

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Superconductor Electronics: Fundamentals and Microwave Applications. 1st edition, Springer.

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SYBILLE HAAS:
"Low Noise Fixed-Tuned SIS Mixers for Astronomical Observations in the Submm Wave Region", 1998

DIRK HOTTGENROTH:
"Superconductor-Insulator-Superconductor Heterodyne Mixers on Niobium basis above the Gap Frequency of Niobium and their use in an Astronomical Receiver", 1997

STEFAN GLENZ:
"Fabrication and Characterization of Nb-Al/Al_2O_3-Nb Superconductor-Insulator-Superconductor Devices with NbTiN Based Tuning Circuits for the HIFI Instrument on the Herschel Space Observatory", 2005

Internet resources:

http://www.astro.uni-koeln.de/sis
https://www.wmi.badw.de/teaching/lecture-notes
Applied Superconductivity

2 Fundamentals

2.1 Fundamental phenomena of superconductivity

During his research of the conductivity of metals at low temperatures, Heike Kamerlingh Onnes discovered in 1911 that the ohmic resistance of mercury disappears at a temperature of about 4.2 K. He realized that a transition of the material into a different physical state took place, which he called "superconductivity".

Once the electrical resistance has disappeared below a certain temperature (the critical temperature T_C), superconductors reveal further effects. For example, magnetic fields are expelled from the superconductor. The external magnetic field induces currents which flow within a thin layer at the surface of the superconductor. These in turn create a magnetic field opposite to the external field. Thus, a superconductor behaves like a perfect diamagnet. This magnetic property of superconductors was discovered in 1933 by Meißner and Ochsenfeld and is therefore called Meissner-Ochsenfeld effect.

2.2 BCS theory

One of the most important findings of the microscopic theory by Bardeen, Cooper and Schrieffer is the fact that superconductivity can be explained by an attractive interaction between two electrons. The attractive effect is established by electrons interacting via phonons while they are moving through the crystal lattice. This interaction is strongest when the momenta and spins of the electrons are of the same magnitude and opposite direction. In this case, the attractive interaction is stronger than the Coulomb repulsion and both the electrons correlate as a pair. The average distance of the electrons of this so-called Cooper pair is called their coherence length ξ_{Co}. There are further 10^6-10^7 electrons within this distance, which are correlated to Cooper pairs as well. Furthermore, BCS theory states that correlation exists not only between the electrons of each Cooper pair, but also between all Cooper pairs. This is why the whole of all Cooper pairs can be expressed by a single wavefunction with a common phase:

$$\psi(x) = |\psi(x)| \cdot e^{i\theta(x)} \qquad (1)$$

The phase coherence of Cooper pairs persists along the whole of the superconductor and leads to the observation of macroscopic quantum effects. Cooper pairs behave like bosons. They are no more restricted by the Pauli principle and therefore can occupy the same ground state. Excited states of the wavefunction are referred to as quasiparticles, which are similar to electrons or holes depending on their character. It is not possible to simply describe them as electrons, because the approximations of the Sommerfeld model for a fermionic gas of non-interacting electrons do not apply here. However, the quasiparticles play the role of charge carriers with roughly one electron charge, similar to a single-electron state. In this tutorial, the term "single-electron state" is to be understood in the sense of "quasiparticles".

One of the key predictions of BCS theory was the formation of a forbidden zone around the Fermi energy where no energy states can be occupied. The width of the energy gap is $E_g = 2\Delta$ and can also be interpreted as the binding energy of the Cooper pairs. This energy is needed to break up a Cooper pair and create two quasiparticle excitations. The single-electron states, which would occupy the forbidden zone in the normal metal state, are squeezed at the boundary of the energy gap, i.e. the density of states of the single electrons increases strongly at its edges.

This can be used to develop a model for quasiparticles and quasiparticle tunneling using a semiconductor model. In that model, the charge carrier density as a function of energy (approximately constant for a normal conductor) is visualized with a "bandgap" in the quasiparticle states caused by the pairing energy. However, it must be considered that many aspects of superconductivity are not covered by this model (such as e.g. the paired electrons!).

The width of the energy gap depends on the temperature. It has its largest value at absolute zero and decreases with increasing temperature. The temperature dependence can be described by:

$$\frac{\Delta(T)}{\Delta(0)} \cong \sqrt{\cos\left[\frac{\pi}{2}\left(\frac{T}{T_C}\right)^2\right]} \qquad (2)$$

In addition, BCS theory yields a relation between the energy gap at 0 K and the critical temperature T_C:

$$2\Delta(0) = 3.52 \cdot k_B T_C \qquad (3)$$

This relation is a good approximation for most classical superconductors (in contrast to high-temperature superconductors).

2.3 Flux quantization

One of the quantum effects that are macroscopically observable due to phase coherence is the quantization of magnetic flux (e.g. in a superconducting ring): The magnetic flux Φ_F through an open surface F with an electric current along its contour can only be an integral multiple of the fluxon Φ_0. As explained in the previous section, the entire ensemble of the Cooper pairs can be described by a single wavefunction (1). When considering the path of the Cooper pairs along a closed line inside the superconductor, it follows from phase coherence that the wavefunction must return to itself after one cycle. The phase θ can thus change only by integral multiples of 2π. This fact may be expressed equivalently as a quantum condition for the momentum of a Cooper pair:

$$\oint \vec{p}_C \cdot \mathrm{d}\,\vec{s} = n h, \text{ with } n = 0,1,2,\ldots \tag{4}$$

After insertion of the canonical momentum $\vec{p} = m_C \vec{v} + 2 e \vec{A}$, with the mass of a Cooper pair $m_C = 2\, m_e$, and rearrangement the resulting equation is:

$$n h = 2 e \left[\oint \frac{m_C}{(2e)^2 n_C} \vec{j}_S \cdot \mathrm{d}\,\vec{s} + \oint \vec{A} \cdot \mathrm{d}\,\vec{s} \right] \tag{5}$$

Herein the velocity was converted into a current density for the number density of Cooper pairs n_C by using $\vec{j}_S = 2 e\, n_C \vec{v}$.

The currents inside a superconductor flow only inside a thin layer at its surface. Therefore, the value of the line integral of the current density along a path inside the superconductor is zero. With $\nabla \times \vec{A} = \vec{B}$ and Stokes' theorem follows:

$$\oint \vec{A} \cdot \mathrm{d}\,\vec{s} = \iint_F \left(\nabla \times \vec{A}\right) \cdot \mathrm{d}\,\vec{F} = \iint_F \vec{B} \cdot \mathrm{d}\,\vec{F} = \Phi_F \tag{6}$$

That implies for the magnetic flux Φ_F:

$$\Phi_F = n \frac{h}{2e} \tag{7}$$

and for the fluxon:

$$\Phi_0 = \frac{h}{2e} \approx 2 \cdot 10^{-11} \frac{\mathrm{T}}{\mathrm{cm}^2} \tag{8}$$

This result for the quantization of magnetic flux was confirmed experimentally by Doll and Näbauer as well as by Deaver and Fairbank in 1961, providing additional support for BCS theory.

2.4 SIS tunnel junctions

SIS tunnel junctions are components consisting of a superconductor–insulator–superconductor layer stack. The insulating barrier, which separates the two superconducting electrodes, has a width of only about a nm. Hence, there is a non-vanishing quantum mechanical probability for the charge carriers to tunnel though the barrier. In this section only the single-electron tunneling process shall be considered. The I–V characteristic of an SIS element is depicted in 1.

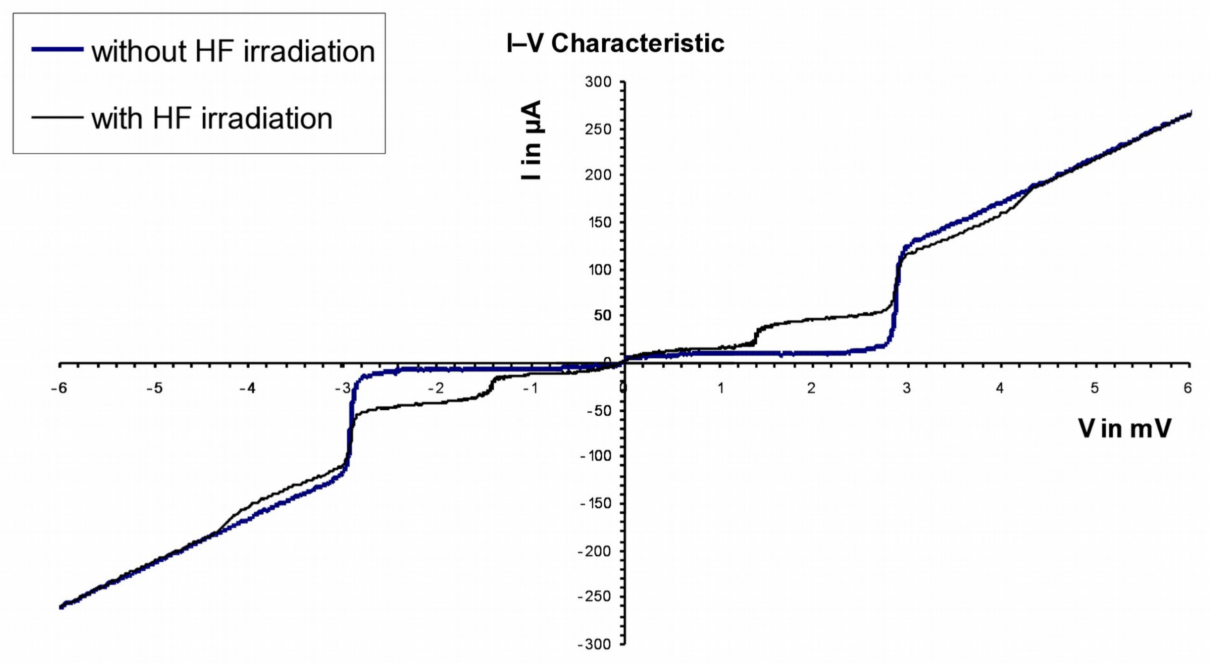

Fig. 1:SIS I–V characteristics, recorded in the KOSMA laboratory

This I–V characteristic can be explained well with the electronic band structure of superconductors in the semiconductor model. When in their ground states, the superconductors are at energetic equilibrium. The single electrons cannot tunnel through the barrier because there are no unoccupied states of same energy in the other superconductor. When an external voltage is applied, one of the superconductors is lowered in energy relative to the other. However, a tunneling process cannot take place until the voltage reaches the value of the energy gap (we are assuming the same superconductors at both sides of the barrier):

$$U_0 = \frac{2\Delta}{e} = U_{gap} \qquad (9)$$

A current will start to flow through the barrier only above this voltage (energy gap voltage). Due to the high densities of state of the charge carriers at the boundaries of the energy gap, the increase in current is exceptionally high. When the voltage increases further, the slope of the I–V characteristic decreases as a consequence of the diminishing density of charge carriers and approaches the resistance in the normal metal state.

A closer inspection of the I–V characteristic reveals that a small current flows through the barrier even below $U_0 = 2\Delta/e$. This current is due to the thermally excited quasiparticles for $T > 0\,\mathrm{K}$.

If an SIS tunnel junction is irradiated with photons, the I–V characteristic shows distinctive steps. The energy level of the charge carriers is then shifted by $n\hbar\omega$, which is equivalent to an applied voltage of $n\hbar\omega/e$. This means that, if a bias voltage U_0 is applied, a current flows corresponding to the current at the voltage $U_0 + n\hbar\omega/e$ of the non-irradiated SIS tunnel junctions. The irradiation thus leads to photon-assisted tunneling processes of the quasiparticles. The photon steps appear in the I–V characteristic at a distance of $\hbar\omega/e$. An increase in current thus occurs at voltages $U_{photon} = U_{gap} - n\hbar\omega/e$ which is an image of the current increase at the gap voltage U_{gap} .The I–V characteristic under photon irradiation is described by the Tien-Gordon equation:

$$I_{TG}(U_0, U_{LO}) = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eU_{LO}}{\hbar\omega}\right) \cdot I_{DC}\left(U_0 + \frac{n\hbar\omega}{e}\right) \qquad (10)$$

U_{LO} is the amplitude of the photon-induced alternating voltage (LO = local oscillator), J_n are the Bessel functions of the 1st kind and I_{DC} is the I–V characteristic of the SIS-Contacts without RF irradiation. Cf. also eqn (19).

2.5 Josephson effects

Besides the quasiparticles, Cooper pairs may tunnel through an SIS tunnel junction as well if the barrier is thin enough (1–2 nm). Cooper pair tunneling and related effects were predicted theoretically by B. D. Josephson and could be corroborated experimentally soon afterwards. Due to the thin barrier, the wavefunctions $\Psi_1(x)$ and $\Psi_2(x)$ of both superconductors are not independent any more, but there is a weak coupling between them. This results in a supercurrent which depends on the phase difference $\Delta\varphi = \varphi_2 - \varphi_1$ between the two wavefunctions.

The two most important properties of the Josephson effects, which are also called Josephson equations, are:

DC Josephson effect: $j = j_C \sin(\Delta\varphi)$ (11)

AC Josephson effect: $\frac{\partial(\Delta\varphi)}{\partial t} = \frac{2eU}{\hbar}$ (12)

The first of both equations describes the current density of the tunnel current, which may penetrate the barrier up to a maximum or critical current density j_C. The critical current density is coupled to the phase difference between the wavefunctions of the two superconductors. The fact that a current flows even at zero voltage when a current source is connected is called the **DC Josephson effect**. Above the critical current, the tunnel junction enters (hysteretically) into the quasiparticle ("single elctron") tunneling range.

The critical current I_C is given by:

$$I_C = \text{SCF} \cdot G_N \cdot \frac{\pi \Delta(T)}{2e} \cdot \tanh \frac{\Delta(T)}{2 k_B T} \qquad (13)$$

SCF is the "strong coupling factor". It characterizes the strength of electron–phonon interaction. For weak coupling $SCF = 1$ and for strong interactions $SCF < 1$ (e.g. SCF = 0.788 for lead). G_N is the electrical conductance in the normal tunneling (linear) range and $\Delta(T)$ the temperature-dependent size of the energy gap.

The second Josephson equation (12) expresses the change of phase difference in time. If the voltage U across the contact is $\neq 0$, the phase difference will not remain constant any more. As a consequence, an alternating current with frequency $\omega_J = 2eU/\hbar$ will flow (**AC Josephson effect**). This arises from integrating eqn (12) and inserting the result in eqn (11). The current density is then:

$$j = j_C \sin(\omega_J t + \varphi_0) \qquad (14)$$

It follows for the dependence of the AC frequency on the applied voltage:

$$\nu_J(U) = \frac{\omega_J(U)}{2\pi} = 483.6 \frac{\text{GHz}}{\text{mV}} \cdot U \qquad (15)$$

2.6 AC-Josephson effect and Shapiro steps

An indirect proof for the AC Josephson effect becomes apparent with the absorption of radio frequency radiation which was already described as the photon assisted tunneling process. If the alternating voltage $U_{LO} \cos(\omega_{LO} t)$ caused by the photon irradiation coupled into the circuit is added to a constant voltage U_0 , the total voltage will be:

$$U(t) = U_0 + U_{LO} \cos(\omega_{LO} t) \qquad (16)$$

Insertion into eqn (12) and integration yields:

$$\Delta\varphi = \frac{2eU_0}{\hbar} t + \frac{2eU_{LO}}{\hbar \omega_{LO}} \sin(\omega_{LO} t) + \varphi_0 \qquad (17)$$

The Josephson current is then:

$$I_J(t)=I_C\sin\left(\frac{2eU_0}{\hbar}t+\frac{2eU_{LO}}{\hbar\omega_{LO}}\sin(\omega_{LO}t)+\varphi_0\right) \quad (18)$$

By using the trigonometric addition theorems and the expansion of sine and cosine into Bessel functions[1] one obtains:

$$I_J(t)=I_C\sum_{n=-\infty}^{\infty}(-1)^n J_n\left(\frac{2eU_{LO}}{\hbar\omega_{LO}}\right)\sin[(\omega_J-n\omega_{LO})t+\varphi_0] \quad (19)$$

with the phase φ_0 and the Bessel functions of the 1st kind J_n.

As one can see from eqn (19), an additional DC component will only appear if the condition $n\omega_{LO}=\omega_J$ is satisfied. In this case, steps occur in the I–V characteristic at the voltages $U_n=n\frac{\hbar\omega_{LO}}{2e}$, that are named Shapiro steps after their discoverer. Note the difference between the Shapiro step voltages U_n and the photon step voltages U_{photon}! With increasing frequency, the voltages of the Shapiro steps increase, whereas the voltages of the photon steps decrease.

2.7 Dependence of the Josephson effects on magnetic fields

An external magnetic field penetrating the tunnel junction parallel to the barrier plane and perpendicular to the direction of current will influence the maximum critical current I_C. The magnetic field brings about a spatial modulation of the phase difference $\Delta\varphi$ along the tunnel junction.

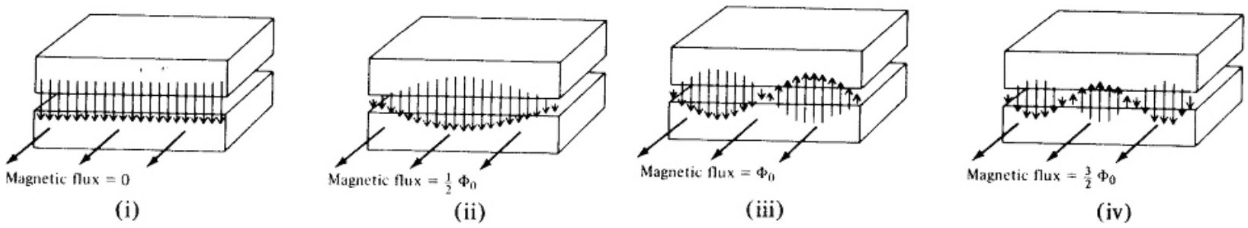

Fig. 2:Modulation of the Josephson current by a magnetic field

The phase difference along the tunnel junction (direction of the *x*-axis) as a function of the magnetic

1 ABRAMOWITZ, M.; STEGUN, I. (1964): Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. U.S. Department of Commerce, National Bureau of Standards

field is found to be:

$$\Delta\varphi(B_0, x) = \frac{2ed'}{\hbar} B_0 \cdot x + \varphi_0 \tag{20}$$

Wherein B_0 is the magnetic field and $d' = d + 2\lambda_L$ the width d of the barrier plus the magnetic (London) penetration depth on both sides of the barrier. The current density is then:

$$j_S(B_0, x) = j_C \sin\left(\frac{2ed'}{\hbar} B_0 \cdot x + \varphi_0\right) \tag{21}$$

It follows for the magnetic field dependence of the maximum critical current flowing through the tunnel junction in case of a rectangular tunnel junction geometry:

$$I_C(\Phi) = I_C(0) \left| \frac{\sin\left(\pi \frac{\Phi}{\Phi_0}\right)}{\left(\pi \frac{\Phi}{\Phi_0}\right)} \right| \tag{22}$$

Φ_0 is the elementary quantum of magnetic flux (fluxon) and $\Phi = d' L B_0$ (with L being the width of the tunnel junction perpendicular to the magnetic field) the magnetic flux which permeates the tunnel junction.

In 3, the critical current $I_C(\Phi)$ is plotted as a function of the magnetic flux. The figure is the same as a single-slit diffraction pattern. The zeroes of $I_C(\Phi)$ are located where the value of magnetic flux is an integral multiple of the fluxon Φ_0.

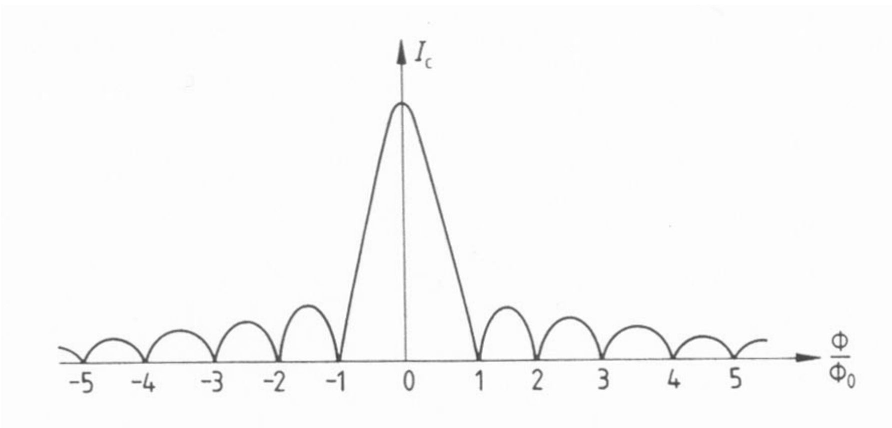

Fig. 3: Dependence of the critical current on the magnetic flux

3 Experimental setup

3.1 Superconducting tunnel junction and mount of instruments

In this experiment, a Nb/AlOx/Nb tunnel junction is used to investigate the I–V characteristic of a superconducting tunnel junction. The SIS tunnel junction is fabricated on a small quartz substrate and integrated into a superconducting high frequency circuit, which is designed for matching the impedance of the SIS junction to the waveguide from which the millimeter wave high frequency signal is coupled in. The substrate is integrated into a (20 x 20 x 10 mm) mixer block. The DC contacts (four-probe sensing) and superconducting magnets for the suppression of the Josephson effects are integrated in the mixer block. The mixer block itself is mounted on a dipstick, which can be dipped into a helium cryostat for the measurements at 4.2 K. The DC contacts are connected with thin wires that run through the dipstick and are connected to the main electronics (rack-mounted bias supply box) via a multi-pole measurement cable. A diode for temperature measurement is installed in the head of the dipstick right next to the mixer block. The RF radiation is generated by a local oscillator (Gunn diode + Schottky diode frequency multiplier) and coupled in with a waveguide.

3.2 Technique of measurement

For the measurement of the I–V characteristic, the dipstick is connected to the bias supply box, which serves mainly as a voltage supply. From here, the measurement signals are directed to the two-channel oscilloscope and in parallel to the PC via an A/D converter (National Instruments). Digital data recording is controlled by a program which displays the I–V characteristic on-line, together with the temperature at the SIS mixer and the applied superconducting magnet current. The computer-recorded data can be stored in ASCII format.

<u>Important</u>: Before instruments are switched on, attention must be paid to the switches on the dipstick box and the "Bias" switch on the electronics box being set to **"off" and "safe"** before the dipstick is connected. Otherwise, the tunnel junction may be damaged.

As all components are extremely sensitive to electrostatic discharge (ESD), you have to wear a grounding strap during mounting operation.

The instruction mentioned below must be followed upon local oscillator startup in the second part of the experiment:

<u>Important</u>: Before the Gunn oscillator is switched on, attention **<u>must</u>** be paid to the adjustable **<u>attenuator</u>** (micrometer screw) between Gunn oscillator and multiplier being **<u>closed</u>**, i.e. being set to maximum attenuation by turning the screw clockwise up to the stop.

The Gunn oscillator operating voltage amounts to 10 volts at 0.25 amperes and is supplied by the bias electronics. After switching on the Gunn oscillator, the attenuator is opened until the effects can be observed at the I–V characteristic.

4 Experimental procedure

4.1 Cooling and warming the dipstick

The tunnel junction is immersed in liquid helium to cool it down to 4.2 K. For this purpose, the dipstick is placed with its compression fitting upon the flange of the helium cryostat, with the dipstick extended up. Please make sure that the dipstick is not lowered too quickly into the helium cryostat during the cooling process in order to reduce helium usage. Besides the compression fitting, an additional clamp serves as a retaining element for the dipstick.

The heating of the dipstick proceeds reverse to its cooling.

The assistant has to be present at both the cooling and the warming process of the sample.

The dipstick has nomagnetic shielding. Sometimes you will see effecxts of external stray fields or trapped flux in the superconductor which (partially) suppress the Josephson current even without applied magnet current. In addition there can be a remanent field in the magnet core. The remant field or the frozen flux can often be compensated by an appropriate opposite setting of the magnet current. You should always perform this compensation whenever a measurement „without magnet ic field“ is asked for, which should rather mean „with the maximum Josephson effect“.

4.2 Measurement of the I–V characteristics

The I–V characteristic shall be observed while the dipstick is lowered. When the critical temperature is reached, the previously linear I–V characteristic changes and takes the typical form of an SIS I–V characteristic. The transition to the superconducting state and the formation of Cooper pairs (Bose–Einstein condensation) can here be observed macroscopically.

4.2.1 I–V characteristics of the tunnel junction without HF irradiation

In this first part of the experiment, several I–V characteristics of the superconducting tunnel junction as a function of temperature shall be measured. Vary the temperature by slowly raising the dipstick in the Helium can. Determine the gap voltage from each quasiparticle I–V characteristic (without magnetic field). Choose the number of measurements and temperatures such that you have good data for a fit to the theory. At the lowest temperature, take one measurement with the Josephson DC current fully suppressed and one with the maximum possible Josephson current visible.

4.2.2 I–V characteristics with RF irradiation

In this part of the experiment, the local oscillator is installed and coupled to the waveguide flange of the dipstick. After switching on the local oscillator and opening the attenuator, “pumped” I–V characteristics shall be recorded. For three different frequencies (tuning of the oscillator by the supervisor) take recordings of I–V characteristics with suppressed Josephson effects, followed by a measurement with the maximum number of Shapiro steps visible (adjust the magnetic field and the local oscillator attenuator accordingly). Take one measurement without RF radiation and use it to determine the gap voltage for all frequency determinations with photon steps (chapter 5.2)

5 Analysis and interpretation

In all diagrams please clearly mark the data points that you read off the diagrams so that your readings can be independently reconstructed. Show zoomed parts if needed.

5.1 I–V characteristics of the tunnel junction without HF irradiation

Sort the I-V data after the voltage and use the current average for measurements with the same voltage value. Calculate the value of the resistance in the normal tunneling range, R_N, from the recorded quasiparticle I–V characteristic at the lowest temperature. From the I–V characteristic recorded without a magnetic field, determine the critical current I_C (use SCF = 0.8) and compare it to the value calculated from eqn (13). How do you explain a possible deviation?

After sufficient smoothing of the data, determine the numerical derivative of the current. Use the maxima of the derivative to determine the gap voltage. Determine the value of $2\Delta(T)$ using the voltages U_{gap}, for each temperature and fit the data to the theoretical curve. Determine $2\Delta_{Niob}(0)$.

in meV and the critical temperature from the fit. Niobium films have a Tc of about 9.2K. Discuss possible reasons if your value differs. Check whether the established value of $2\Delta_{Niob}(0)$ fits the prediction for the energy gap made by BCS theory.

5.2 I–V characteristics with HF irradiation

Determine the voltage of the photon steps and the voltage of the Shapiro steps from the I–V characteristics recorded with RF irradiation and use the result to obtain the three RF frequencies, separate for the two types of steps. Use again the maxima of the numerical derivative of the I-V curves to determine the step voltages. Carefully separate the Shapiro steps from the photon steps in the measurements without magnetic field. This takes some effort as the some of the steps are close together or even overlap. Use the knowledge about the photon steps taken from the curves with magnetic field. Clearly mark the photon steps and the Shapiro steps in the diagrams and add the order n. Discuss your results. Why do the Shapiro steps of order n=2 appear so close (or even at the same voltage) to the photon step voltage with n=1?

Optional bonus task: try a fit of the Tien-Gordon equation to one of the pumped I/V's with suppressed Josephson current. Fit parameters are U_{LO} and the frequency ω.